Technical Notes

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Calculation of Shape Factors Between Parallel Ring Sectors Sharing a Common Centerline

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Nomenclature

x,y,z	= cartesian coordinates of dA_1, cm
ℓ_1, m_1, n_1	= cosines (i.e., direction cosines) of angles between normal to dA_1 and x-, y-, and z- axes, respectively
x_2, y_2, z_2	= cartesian coordinates of point on periphery of surface 2, cm
s	= distance between dA_1 and point on periphery of surface 2, cm
r_0, r_i	= outer and inner radii, respectively, of surface 2, cm
r	= radial coordinate in plane of surface 2, cm
h	= vertical distance between surfaces 1 and 2, cm
ρ	= radial coordinate in plane of dA_1, cm
β	= angular displacement of dA_1 with respect to x-axis, radians
β_1, β_2	= angular displacements which define location of surface 1 with respect to x-axis, radians
θ_1, θ_2	= angular displacements which define location of surface 2 with respect to x-axis, radians
ω	$= \theta - \beta$, radians
ω_1	$= \theta_1 - \beta$, radians
ω_2	$= \theta_2 - \beta$, radians

Introduction

IN thermal control calculations for spin-stabilized spacecraft, it is often desired to calculate the shape factor for diffuse radiant-energy exchange between parallel ring sectors sharing a common centerline. If the ring sectors span 360° of arc, calculation of the shape factor can be accomplished by means of shape-factor algebra and the closed-form expression for the shape factor between parallel disks.[1] A comparable expression for ring sectors spanning less than 360° of arc is not readily available in the literature.

The contour integral method is used in the present analysis to derive a closed-form expression for the shape factor from a differential area on one ring sector to the other (finite-sized) ring sector. This expression can then be integrated numerically to obtain the desired shape factors between two finite-sized ring sectors. Typical results are shown for both configurations.

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Analysis

Consider the geometry shown in Fig. 1, which illustrates the nomenclature used in this presentation. Throughout this discussion, surface 2 (A_2) is always considered to be of finite size; surface 1 can be either a differential area (dA_1) or a finite-sized area (A_1), depending on the problem under consideration.

Sparrow[2] has shown that the shape factor, $F_{dA_1-A_2}$, can be expressed as the sum of three contour integrals in the following manner:

$$F_{dA_1-A_2} = \ell_1 \int_C \frac{(z_2-z)dy_2-(y_2-y)dz_2}{2\pi s^2} + m_1 \int_C \frac{(x_2-x)dz_2-(z_2-z)dx_2}{2\pi s^2} + n_1 \int_C \frac{(y_2-y)dx_2-(x_2-x)dy_2}{2\pi s^2} \quad (1)$$

where

$$s^2 = (x_2-x)^2+(y_2-y)^2+(z_2-z)^2 \quad (2)$$

The letter C designates integration around the periphery of surface 2. Since the normal to dA_1 is parallel to the z-axis, $\ell_1 = m_1 = 0$, and $n_1 = 1$. Hence, only the last term in Eq. (1) is nonzero.

Integration around the periphery of surface 2 is performed in the clockwise direction in four steps, each step corresponding to one of the numbered boundaries shown in Fig. 1. For boundary 1: $x_2 = r_i \cos\theta$, $dx_2 = -r_i \sin\theta d\theta$, $y_2 = r_i \sin\theta$, $dy_2 = r_i \cos\theta d\theta$. For boundary 2: $x_2 = r\cos\theta_2$, $dx_2 = \cos\theta_2 dr$, $y_2 = r\sin\theta_2$, and $dy_2 = \sin\theta_2 dr$. For boundary 3: $x_2 = r_0\cos\theta$, $dx_2 = -r_0\sin\theta d\theta$, $y_2 = r_0\sin\theta$, and $dy_2 = r_0\cos\theta d\theta$. For boundary 4: $x_2 = r\cos\theta_1$, $dx_2 = \cos\theta_1 dr$, $y_2 = r\sin\theta_1$, and $dy_2 = \sin\theta_1 dr$. For all boundaries, $x = \rho\cos\beta$ and $y = \rho\sin\beta$.

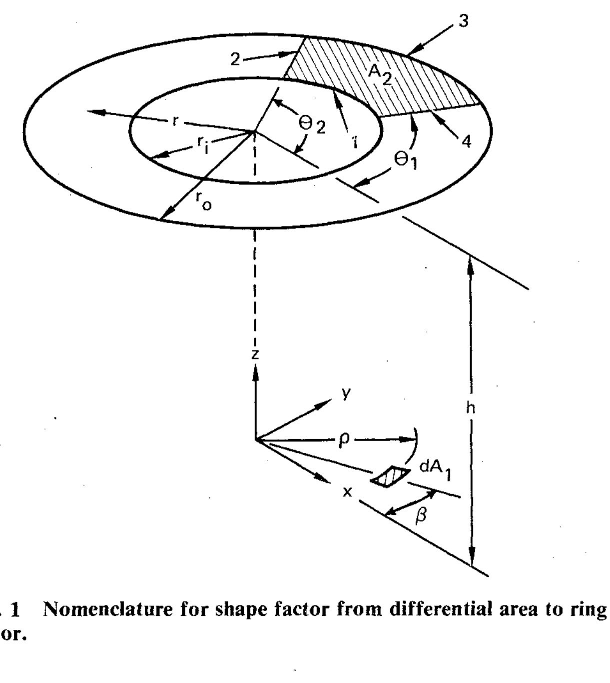

Fig. 1 Nomenclature for shape factor from differential area to ring sector.

Substituting these expressions for x, y, x_2, and y_2 into Eq. (1) and making use of the identities

$$\cos(\omega)=\cos(\theta-\beta)=\sin(\beta)\sin(\theta)+\cos(\beta)\cos(\theta)$$
$$\sin(\omega)=\sin(\theta-\beta)=\cos(\beta)\sin(\theta)-\sin(\beta)\cos(\theta) \tag{3}$$

results in the expression

$$F_{dA_1-A_2}=-\frac{r_i}{2\pi}\int_{\omega_1}^{\omega_2}\frac{[r_i-\rho\cos(\omega)]\,d\omega}{r_i^2+\rho^2+h^2-2r_i\rho\cos(\omega)}-\frac{r_0}{2\pi}\int_{\omega_2}^{\omega_1}\frac{[r_0-\rho\cos(\omega)]\,d\omega}{r_0^2+\rho^2+h^2-2\rho r_0\cos(\omega)}$$
$$+\frac{\rho\sin(\omega_2)}{2\pi}\int_{r_i}^{r_0}\frac{dr}{r^2+h^2-2\rho r\cos(\omega_2)+\rho^2}+\frac{\rho\sin(\omega_1)}{2\pi}\int_{r_0}^{r_i}\frac{dr}{r^2+h^2-2\rho r\cos(\omega_1)+\rho^2} \tag{4}$$

The integrals in this expression are available in standard references.[3] The desired expression for $F_{dA_1-A_2}$ is then found to be

$$2\pi F_{dA_1-A_2}=\frac{\rho\sin(\omega_1)}{[h^2+\rho^2\sin^2(\omega_1)]^{1/2}}\left\{\tan^{-1}\left[\frac{r_i-\rho\cos(\omega_1)}{[h^2+\rho^2\sin^2(\omega_1)]^{1/2}}\right]-\tan^{-1}\left[\frac{r_0-\rho\cos(\omega_1)}{[h^2+\rho^2\sin^2(\omega_1)]^{1/2}}\right]\right\}$$
$$+\frac{\rho\sin(\omega_2)}{[h^2+\rho^2\sin^2(\omega_2)]^{1/2}}\left\{\tan^{-1}\left[\frac{r_0-\rho\cos(\omega_2)}{[h^2+\rho^2\sin^2(\omega_2)]^{1/2}}\right]-\tan^{-1}\left[\frac{r_i-\rho\cos(\omega_2)}{[h^2+\rho^2\sin^2(\omega_2)^{1/2}]}\right]\right\}$$
$$+\frac{\rho^2+h^2-r_i^2}{[(r_i^2+\rho^2+h^2)^2-(2r_i\rho)^2]^{1/2}}\left\{\tan^{-1}\left[\frac{[(r_i^2+\rho^2+h^2)^2-(2r_i\rho)^2]^{1/2}\tan(\omega_2/2)}{r_i^2+\rho^2+h^2-2r_i\rho}\right]-\tan^{-1}\left[\frac{[(r_i^2+\rho^2+h^2)^2-(2r_i\rho)^2]^{1/2}\tan(\omega_1/2)}{r_i^2+\rho^2+h^2-2r_i\rho}\right]\right\}$$
$$+\frac{(\rho^2+h^2-r_0^2)}{[(r_0^2+\rho^2+h^2)^2-(2r_0\rho)^2]^{1/2}}\left\{\tan^{-1}\left[\frac{[(r_0^2+\rho^2+h^2)^2-(2r_0\rho)^2]^{1/2}\tan(\omega_1/2)}{r_0^2+\rho^2+h^2-2r_0\rho}\right]-\tan^{-1}\left[\frac{[(r_0^2+\rho^2+h^2)^2-(2r_0\rho)^2]^{1/2}\tan(\omega_2/2)}{r_0^2+\rho^2+h^2-2r_0\rho}\right]\right\} \tag{5}$$

Consider the special case where $\rho=r_i=0$. For this situation, Eq. (5) reduces to

$$F_{dA_1-A_2}=\left(\frac{r_0^2}{r_0^2+h^2}\right)\left(\frac{\theta_2-\theta_1}{2\pi}\right) \tag{6}$$

For $\theta_2-\theta_1=2\pi$, Eq. (6) reduces to the familiar expression for the shape factor from a differential area to a disk, where the differential area lies on the disk centerline.

Now consider the case where $\omega_1=0$, $\omega_2=2\pi$, and $r_i=0$. For this configuration, it is seen that Eq. (5) reduces to

$$F_{dA_1-A_2}=\frac{1}{2}\left[1+\frac{r_0^2-\rho^2-h^2}{[(r_0^2+\rho^2+h^2)^2-(2\rho r_0)^2]^{1/2}}\right] \tag{7}$$

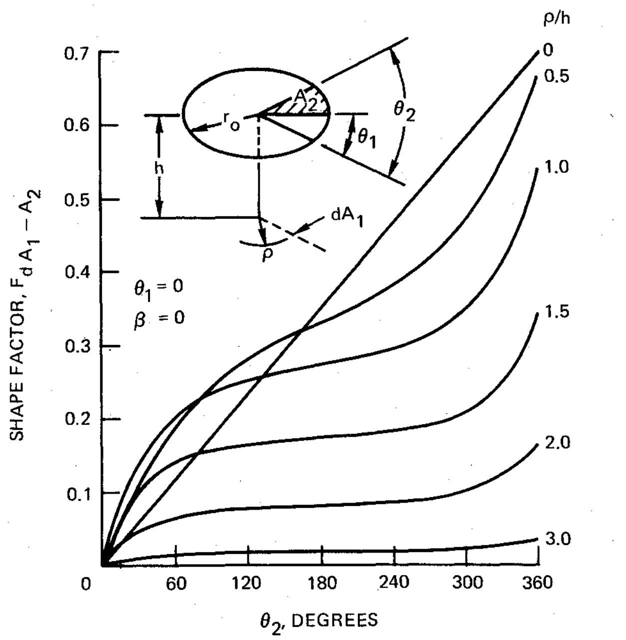

Fig. 2 Shape factor from differential area to disk sector ($r_0/h=1.5$).

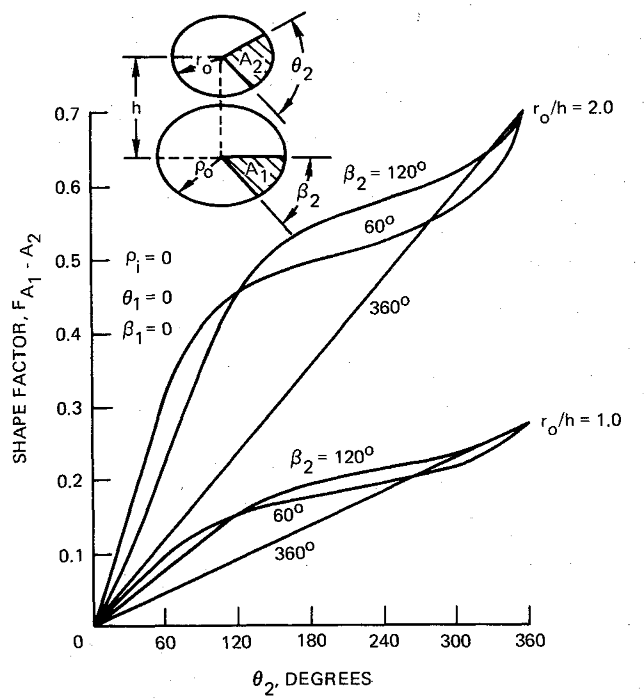

Fig. 3 Shape factor between two finite-sized ring sectors ($\rho_0/h=1.5$).

which is the familiar expression for the shape factor from an off-center differential area to a disk.[1] Note that for $\omega>\pi$, care must be taken in evaluating the inverse tangents in the last two terms of Eq. (5).

Typical shape factors from a differential area to a disk sector ($r_i=0$) are shown in Fig. 2. Here, θ_1 and β were taken as zero, and the shape factors are plotted against θ_2 for several values of ρ/h. In all cases, when $\rho/h=0$ the shape factor is a linear function of θ_2, as shown previously in Eq. (6).

Shape factors between two finite-sized ring sectors can be determined by integrating Eq. (5) in the following manner:

$$F_{A_1-A_2}=\frac{2}{(\beta_2-\beta_1)(\rho_0^2-\rho_i^2)}\int_{\rho_i}^{\rho_0}\rho\left[\int_{\beta_1}^{\beta_2}F_{dA_1-A_2}\,d\beta\right]d\rho \tag{8}$$

where $\beta_2 > \beta_1$ and $\rho_0 > \rho_i$.

Evaluation of Eq. (8) in closed form is at best difficult. Therefore, Eq. (8) was evaluated numerically, and some typical results are shown in Fig. 3. Note that at $\theta_2 = 360°$ (θ_1 assumed to equal zero), the value of $F_{A_1-A_2}$ is the same as that for two fully circular disks regardless of the values of β_1 and β_2. Also note that for $\beta_2 - \beta_1 = 360°$, $F_{A_1-A_2}$ is a straight line.

References

[1] Hamilton, D.C. and Morgan, W.R., "Radiant-Interchange Configuration Factors," NACA TN 2836, 1952, pp. 37, 42.

[2] Sparrow, E.M., "A New and Simpler Formulation for Radiative Angle Factors," *Transactions of the ASME, Series C: Journal of Heat Transfer,* Vol. 85, 1963, pp. 81-88.

[3] Gradshteyn, I.S. and Ryzhik, I.M., *Table of Integrals, Series, and Products,* 4th ed, Academic Press, New York, 1965, pp. 68, 148.

Finite Contact of an Inflated Ring

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Introduction

THE numerical solution is given for a ring which is a) inflated, b) uniform and homogeneous, c) initially circular, and d) under finite (large) compression between rigid, flat, parallel plates (see Fig. 1). The uninflated ring and the inflated cylindrical membrane have been treated.[1,2] For combined inflation and bending, an integral was derived in Ref. 3 with limits to be found from several simultaneous nonlinear algebraic equations. Numerical results were reported only for small deviations from the uninflated and the membrane cases.

In the present full numerical solution, the transition from bending-dominated behavior at low pressure to membrane-dominated behavior at high pressure is studied. Also, we assess an approximate solution[2] in which the force at a given deflection is assumed to consist of independent bending and membrane contributions.

Analysis

1. Nomenclature and Governing Equations

Let X and Y denote the extrinsic coordinates of a point on the ring; denote the pressure as P (positive outward), and let M, V, and T, respectively, denote bending moment (positive anticlockwise), shear force (positive outward), and tension (positive clockwise). Also, R_0 is the initial radius of the ring. Owing to symmetry, attention will be confined to the first quadrant. Also, a unit width in the axial direction is assumed.

The analysis will be in terms of intrinsic coordinates ϕ and s, defined such that $\tan\phi = dY/dX$ where $dX = ds\cos\phi$. As an elastica the ring obeys

$$M = -D[(d\phi/ds) - (d\phi_0/ds)] \tag{1}$$

where the zero subscript refers to the original configuration, and D is the bending rigidity (assumed constant). The quantity $(d\phi/ds)^{-1}$ is the radius of curvature.

We introduce the dimensionless quantities $x = X/R_0$, $y = Y/R_0$, $\sigma = S/R_0$, $m = MR_0/D$, $v = VR_0^2/D$, $t = TR_0^2/D$, and $p = PR_0^3/D$. The required geometric and equilibrium relations are[1]

$$dx/d\sigma = \cos\phi \tag{2a}$$

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$$dy/d\sigma = \sin\phi \tag{2b}$$

$$dv/d\sigma = p - tz \tag{2c}$$

$$dt/d\sigma = vz \tag{2d}$$

$$dz/d\sigma = -V \tag{2e}$$

where z is defined by

$$d\phi/d\sigma = z \tag{2f}$$

2. Boundary Conditions and Solution Method

For loads below a critical value, say F^*, the initial contact point and its neighboring points have infinite radii and belong to the now finite contact zone. Boundary conditions are given below.

For point contact, at $\sigma = 0$,

$$\phi = 0, \quad z = z_0 \tag{3a}$$

where z_0 is specified. At $\sigma = \pi/2$

$$\phi = \frac{\pi}{2}, \quad v = 0, \quad y = 0 \tag{3b}$$

It remains to determine v and y at $\sigma = 0$. Denote these unknowns as $\bar{f}/2$ and $L = \bar{L}/R_0$. If these quantities were known, the foregoing would reduce to an initial value problem with initial conditions at $\sigma = 0$

$$\phi = 0, \quad z = z_0, \quad v = \bar{f}/2, \quad t = pL, \quad y = L \tag{4}$$

For finite contact, denoting $s_1 = R_0\sigma_1$ as the contact half-width, at $\sigma = \sigma_1$

$$\phi = 0, \quad z = 0 \tag{5a}$$

at $\sigma = \pi/2$

$$\phi = \pi/2, \quad v = 0, \quad y = 0 \tag{5b}$$

In finite contact, the total (dimensionless) compressing force on the ring is

$$f = f_b + 2p\,\sigma_1 \tag{6a}$$

where f_b is a concentrated force, interpreted as the bending contribution, acting at σ_1. For point contact, the total force is

$$f = \bar{f} \tag{6b}$$

and hereafter the overbar will not be displayed.

For brevity we discuss only the numerical solution for the point contact stage. Equation (2) together with boundary conditions Eq. (3) defines a two-point boundary value problem which we have solved by the widely studied "shooting" techniques.[4] The auxiliary initial value problem [involving Eq. (4)] was integrated using Hamming's method, and the assumed values of f and L were adjusted iteratively using Newton's method, in order to accommodate all of the boundary conditions. An extrapolation procedure was used

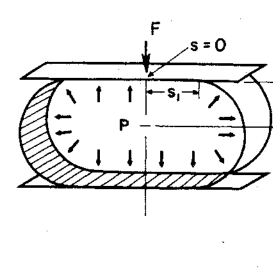

Fig. 1 Inflated ring under compression.